ALGEBRA 1
INTERACTIVE NOTEBOOK
CHAPTER 6: SYSTEMS OF LINEAR EQUATIONS & INEQUALITIES

Math with Sister in Room 2³
What is a System of Linear Equations?

Solution to a Linear Equation:

\[
\begin{align*}
y &= -\frac{2}{3}x + 3 \\
y &= x - 2
\end{align*}
\]

Solution to a System of Linear Equations:

\[
\begin{align*}
y &= -2x - 3 \\
6x + 3y &= -9
\end{align*}
\]

\[
\begin{align*}
2x - y &= -1 \\
4x - 2y &= 6
\end{align*}
\]
consistent and independent systems of equations have exactly __________ because the graphs __________ at exactly one point.

consistent and dependent systems of equations have __________ number of __________ because the graphs are the __________ line.

inconsistent systems of equations have __________ because the graphs are __________ lines.

Types of Solutions to System of Linear Equations:
Today we will:
- Solve systems of linear equations by Substitution.
- Determine the number of solutions a system of linear equations has.

### How do I solve a System of Linear Equations using substitution?

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Solve one equation for one of the variables.</td>
<td>$y = 4x - 6$</td>
</tr>
<tr>
<td></td>
<td>$5x + 3y = -1$</td>
</tr>
<tr>
<td>2) Substitute the expression found into the other equation for that variable; solve for the other variable</td>
<td></td>
</tr>
<tr>
<td>3) Substitute the value for the solved variable into the equation found in Step 1 and solve.</td>
<td></td>
</tr>
<tr>
<td>Example</td>
<td>Example</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>[-2x + y = -4]</td>
<td>[x - 3y = -9]</td>
</tr>
<tr>
<td>[-6x + 3y = -12]</td>
<td>[5x - 2y = 7]</td>
</tr>
</tbody>
</table>

Cut out this chart into two strips and tape as a book on top of the right hand side of the chart on pg. 4.
**How do I solve a System of Linear Equations using Elimination?**

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
</table>
| 1) Consider the two given equations and determine which variable might be eliminated by adding the two equations together. | \[ x - y = 11 \]
<p>|                                                                       | [ 2x + y = 19 ] |
| 2) Add the two equations together and solve for the variable that did NOT get eliminated. |               |
| 3) Substitute the value for the solved variable into the equation found in Step 1 and solve for the second variable. |               |</p>
<table>
<thead>
<tr>
<th>Example</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8x - 5y = 5$</td>
<td>$-x + y = 1$</td>
</tr>
<tr>
<td>$-8x - 6y = 6$</td>
<td>$-2x + y = -4$</td>
</tr>
</tbody>
</table>

Cut out this chart into two strips and tape as a book on top of the right hand side of the chart on pg. 6.
Today we will:
• Solve systems of linear equations by elimination involving:
  - Multiplication.

What are opposite terms?

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
</table>
| 1) Consider the two given equations and determine which variable might be eliminated. Multiply at least one equation by a constant to get the two equations that contain opposite terms. | $6a + 2b = 2$
<p>|                                                                     | $4a + 3b = 8$      |
| 2) Add the two equations together, eliminating one variable. Then solve the equation.          |                    |
| 3) Substitute the value from step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair. |                    |</p>
<table>
<thead>
<tr>
<th>Example</th>
<th>Example</th>
</tr>
</thead>
</table>
| $6x - 2y = 10$  
$3x - 7y = -19$ | $9x + y = 13$  
$3x + 2y = -4$ |

Cut out this chart into two strips and tape as a book on top of the right hand side of the chart on pg. 8.
Aaaa! I can't do these math problems! Why am I so stupid?!

"If 2x + y = 60 and x + 2y = 75, solve for x and y. How the heck do I figure that out?"

If two shirts and a sweater costs $60, and a shirt and two sweaters costs $75, what does each item cost?

The shirts are $15 and the sweaters are $30. Duh.

You aren't stupid, Paige, just weird.

Come back. You still haven't told me how to solve the problem!
How do I solve a System of Linear Equations in the Form of Word Problems?

**FLIGHT.** A personal aircraft traveling with the wind flies 520 miles in 4 hours. On the return trip, the airplane takes 5 hours to travel the same distance. Find the speed of the airplane if the air is still.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Create a TABLE to organize the data in the word problem.</td>
<td></td>
</tr>
<tr>
<td>2) Identify the variables and translate the table into a system of equations</td>
<td></td>
</tr>
<tr>
<td>3) Solve the system.</td>
<td></td>
</tr>
</tbody>
</table>
Today we will:
- Determine the best method for solving systems of equations.
- Apply systems of equations.

### How do I solve a System of Linear Equations in the Form of Word Problems?

**BUSINESS.** Suppose you start a business assembling and selling motorized scooters. It costs you $1500 for tools and equipment to get started, and the materials cost $200 for each scooter. Your scooters will sell for $300 each.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Define your variables and create a system of equations representing the total cost and revenue for your business.</td>
<td></td>
</tr>
<tr>
<td>2) Describe what the solution means in terms of the situation.</td>
<td></td>
</tr>
<tr>
<td>3) What is a reasonable number of scooters you could assemble and sell in order to make a profit, and find the profit you would make for that number of scooters.</td>
<td></td>
</tr>
</tbody>
</table>
Today we will:
• Determine the best method for solving a system of linear equations.
• Apply Systems of Linear Equations.

### Summary of Systems of Linear Equations

<table>
<thead>
<tr>
<th>EXAMPLE</th>
<th>BEST METHOD FOR SOLVING</th>
<th>WHY</th>
</tr>
</thead>
</table>
| 1: $y = -\frac{4}{3}x + 8$  
$y = \frac{4}{3}x$ | | |
| 2: $x = 3$  
$3x + 2y = 5$ | | |
| 3: $3x - 4y = -5$  
$-3x + 2y = 3$ | | |
| 4: $5x + 7y = 2$  
$-2x + 7y = 9$ | | |
| 5: $2x + 3y = -11$  
$-8 - 5y = 9$ | | |
NAME:

Today we will:
- Organize data in matrices.
- Perform matrix operations.

---

**What is a matrix?**

**How do I determine the dimensions of a matrix?**

<table>
<thead>
<tr>
<th>MATRIX VOCABULARY</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEMENT</td>
<td>IDENTITY MATRIX</td>
</tr>
<tr>
<td>ZERO MATRIX</td>
<td>SQUARE MATRIX</td>
</tr>
<tr>
<td>ROW MATRIX</td>
<td>COLUMN MATRIX</td>
</tr>
</tbody>
</table>

**How do I identify each element in a matrix?**

---

6-6 Organizing Data Using Matrices
Today we will:
- Organize data in matrices.
- Perform matrix operations.

Organizing Data into a Matrix

At a meet, 10 points were awarded for each first-place finish, 8 points for each second place finish, and 5 points for each third-place finish. Organize this data from the table below into a matrix.

<table>
<thead>
<tr>
<th>School</th>
<th>Freestyle</th>
<th>Backstroke</th>
<th>Breast Stroke</th>
<th>Butterfly</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>South</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Jefferson</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
NAME:

Today we will:
• Organize data in matrices.
• Perform matrix operations.

6-7
Using Matrices
to Solve
Systems of
Equations

Augmented Matrix

\[
\begin{align*}
x - 3y &= 8 \\
-9x + 2y &= -4
\end{align*}
\]

Write this system of equations as an augmented matrix.

Row Reduction

Identity Matrix

Reduced Row Echelon Form

How do I do this on my calculator?
Today we will:
- Solve and graph linear inequalities with two variables.

**How do I solve a system of linear inequalities?**

1. **Step 1:** Rewrite each inequality in slope-intercept form.

2. **Step 2:** Use the slope and y-intercept of the first equation to graph the first line. Determine whether or not the line needs to be solid or dotted.

3. **Step 3:** Test a point above or below your line to determine which side of the line makes the inequality true. Shade that side of the line.

4. **Step 4:** Use the slope and y-intercept of the second equation to graph the second line.

5. **Step 5:** Test a point above or below your line to determine which side of the line makes the inequality true. Shade that side of the line.

6. **Step 6:** Locate the area on the graph where the entire region is shaded for both inequalities. This is the solution to the system of inequalities.

Sketch the solution to each system of inequalities:

1. \[ y \leq -x - 2 \]
   \[ y \geq -5x + 2 \]

2. \[ y > -x - 2 \]
   \[ y < -5x + 2 \]
NAME:

Today we will:
- Solve and graph linear inequalities with two variables using a TI-84.